The Service Curve Service Discipline with the Service Curve Service Discipline for the Rate-Controlled EDF Service Discipline in Variable-Sized Packet Networks

Kihyun Pyun and Junehwa Song and Heung-Kyu Lee
Department of Electrical Engineering and Computer Science, KAIST
373-1 Kusong-dong, Yusong-gu, Taejon 305-701, Republic of Korea

Abstract—Guaranteed service will provide high quality services to real-time applications, e.g., audio or video, over packet networks such as the Internet. To support guaranteed service, a service discipline must guarantee a delay bound to each session. In addition, a preferred service discipline should achieve high network utilization and good scalability. The service disciplines studied so far have problems in achieving these two objectives at the same time. Generalized Processor Sharing (GPS) service disciplines can have low network utilization. Rate-Controlled (RC) service disciplines have difficulty in scalability because of regulators. For Service Curve (SC) service disciplines, both the network utilization and the scalability depend on the adopted SC. To date, there have been no studies on an SC which can make an SC discipline achieve these two objectives. We propose a new service discipline based on SC service disciplines. The proposed discipline achieves these two goals in a variable-sized packet environment. We show that the discipline can achieve the network utilization achievable by the RC service disciplines. We further show that our SC requires $O(1)$ complexity for deadline calculation. Different from the RC service disciplines, the SC service discipline with our SC does not need regulators at all. Thus, it has better scalability than the RC service disciplines and is work-conserving. We also show that the proposed SC makes SC service disciplines have strictly higher network utilization than the GPS service disciplines including the multi-rate service discipline.

I. INTRODUCTION

Guaranteed service will provide high quality services to real-time applications such as audio/video transmissions over packet networks such as the Internet [15]. To support guaranteed service, each router employs a service discipline. Here, a service discipline refers to an aggregation of a packet scheduler and other required elements to provide a guaranteed service. Such a service discipline should guarantee a delay bound as well as bandwidth to each session at all the output ports. An important aspect of a preferred service discipline is the level of network utilization it can achieve. In addition, a good service discipline should be scalable to support a high number of concurrent sessions.

Service disciplines studied so far have problems with either network utilization or scalability. Generally, a service discipline can achieve high network utilization when it can provide different delay bounds to a given traffic specification. A service discipline has good scalability if its implementation cost and scheduling complexity are not much dependent on the number of sessions. There have been three major trends for service disciplines that can support guaranteed service in the literature: Generalized Processor Sharing (GPS) service disciplines [11], [10], [1], [2], [8], [16], [13], Rate-Controlled (RC) service disciplines [21], [4], [7], and Service Curve (SC) service disciplines [3], [18], [14]. (A more detailed survey is given in Section I-A.) In the GPS service disciplines, delay bounds are decided by specified traffic rates. Thus, achievable network utilization can be very low [7]. Among the RC service disciplines, the RC-EDF service discipline achieves the highest network utilization. However, it has problems in scalability because regulators are required for each session [20], [7]. In the case of the SC service disciplines, network utilization and scalability depend on the adopted service curve. In the literature, so far, there have been no studies on an SC that can make an SC service discipline achieve both high network utilization and good scalability.

In this paper, we propose a new service discipline that can achieve both high network utilization and scalability. The proposed discipline can be used in a variable-sized packet network environment, such as the Internet. It is a special case of an SC service discipline [14]. Devising an SC which can achieve both scalability and high network utilization is not easy. We can achieve high scalability using a simple (e.g., linear) service curve. However, this approach usually results in very low network utilization. On the contrary, if we adopt a complex service curve to achieve high network utilization, scalability becomes a problem since it requires high complexity in deadline calculation. We propose a new SC, called SC-EDF\(^1\) that allows the service discipline to achieve both objectives.

An SC-EDF for a session is constructed from the output function of the regulator of the session and the delay bound at the packet scheduler of the RC-EDF discipline. We show that SC-EDF can make the SC service discipline achieve the network utilization which the RC-EDF discipline can. We further show that SC-EDF becomes piece-wise concave linear in a range of our interest. Therefore, the process for deadline calculation can be simplified requiring only $O(1)$ complexity. Moreover, different from the RC-EDF service discipline, regulators are not required. Therefore, scalability is significantly improved.

Another advantage of the proposed scheme over the RC-EDF service discipline is that it is work-conserving. A service discipline is said to be non-work-conserving if some packets cannot be transmitted to the link although packets are in the service discipline. Generally, a non-work-conserving service discipline makes the average delay of packets larger than that in the work-conserving service disciplines. Regulators make the RC service disciplines non-work-conserving.

\(^1\)The term SC-EDF indicates a new SC proposed in this paper. However, we also use the same term to indicate the SC service discipline which uses the SC-EDF.
Lastly, compared to the GPS service disciplines including the multi-rate service discipline [13], the proposed discipline can achieve strictly higher network utilization.

A. Related work

1) GPS disciplines: The GPS service disciplines such as GPS [11], [10], WF^2Q [2], H-PFQ [1], SCFQ [8], and Rate-Proportional Servers [16] are rate-based service disciplines. These disciplines are different from one another in terms of fairness. They provide the same delay bound to the same traffic rate, and hence, achievable network utilization can be very low.\(^2\)

A traffic rate can be specified either by a constant rate over all intervals or by different rates over different intervals. The traffic rate specified by a constant rate is called a single-rate and that specified by multiple rates is called a multi-rate. The multi-rate service discipline is a GPS service discipline that guarantees a multi-rate to each session [13]. (In the context of scheduling methodology, the multi-rate discipline may look somewhat similar to the SC-EDF [14]. We go through a detailed comparison with SC-EDF in Section IV.) The multi-rate service discipline can achieve higher network utilization than the single-rate GPS service disciplines. However, note that the delay bounds to the same multi-rate are identical. Thus, achievable network utilization can still be low. In all GPS disciplines, including the multi-rate discipline, scalability is dependent on the scheduling complexity, which is \(O(\log N)\) to maintain a sorted priority queue where \(N\) is the number of sessions. Also, the disciplines are work-conserving. Compared to SC-EDF, all GPS service disciplines have similar scalability and lower network utilization.

2) RC disciplines: An RC service discipline [21], [20], [4], [7] consists of regulators and a packet scheduler. It can adopt a different packet scheduler such as a Static-Priority (SP) scheduler or an Earliest Deadline First (EDF) scheduler. Network utilization and scalability are affected by the scheduler. The RC service discipline adopting the SP/EDF scheduler is called an RC-SP/RC-EDF service discipline.

Consider the case of the RC-EDF service discipline [4], [7]. One regulator is required for each session. Each regulator controls the output rate of the corresponding session and stamps a deadline on each incoming packet. The deadline becomes the departure time plus the associated delay bound at the EDF scheduler. The EDF scheduler transmits packets in an increasing order of the deadlines. The RC-EDF service discipline can achieve high network utilization since it can provide different delay bounds to the same traffic rate. However, the RC-EDF service discipline has problems in scalability mainly due to the cost of implementing regulators. (It requires \(N\) regulators, one for each session. Implementing a regulator can require significant overhead for buffer space and a timer that can be expensive.) In addition, the scheduler requires \(O(\log N)\) scheduling complexity. In [20], a method using a calendar queue has been proposed to reduce the cost of implementing separate regulators for each session. However, even in that case, the movements between the calendar queue and the EDF scheduler require \(O(N)\) complexity.

In the case of the RC-SP discipline, the SP scheduler maintains a certain number of priority groups in advance. A session is assigned to a priority group during admission control. The SP scheduler transmits a packet in the highest priority group that has packets in the queue. Compared to RC-EDF, RC-SP has advantage in scalability if a calendar queue method is used [21]. Since the number of priority groups is usually a small constant, the movements between the calendar queue and the SP scheduler require \(O(1)\) complexity. However, the RC-SP service discipline can suffer from low network utilization [9]. Both RC-EDF and RC-SP are non-work-conserving due to the regulators in the disciplines.

Compared to the proposed SC-EDF, RC-SP has better scalability and lower network utilization and RC-EDF has worse scalability and the same level of network utilization.

3) SC disciplines: The SC service disciplines such as SCED [3], [14] or H-FSC [18] are deadline-based disciplines and guarantee an SC for each session. The service curve can be any non-decreasing function. In the SC disciplines, different level of network utilization can be achieved, depending on the selected SC’s for sessions. If the SC’s are constructed by considering only traffic rates, the SC service disciplines can achieve the same level of network utilization as the GPS. If the SC’s are constructed by considering both delay bounds and traffic rates, the SC disciplines can achieve the same level of network utilization as the RC-EDF. When an SC of an arbitrary shape is assumed, the SC disciplines can achieve the highest network utilization even known. The level of scalability is also dependent on the selection of an SC. An SC of an arbitrary shape makes the deadline calculation more complex. Apart from the deadline calculation, the disciplines also require \(O(\log N)\) for scheduling. The SC service disciplines are work-conserving. Most research on SC service disciplines has been performed in the context of a restricted, fixed-sized environment that is applicable to ATM networks.

A general framework for SC service disciplines was proposed in [14]. Our work differs from the work in [14] in the followings. First, our work considers a variable-sized packet environment whereas the work in [14] considered a fixed-sized packet environment. Thus, the results of our work are applicable to packet networks such as the Internet. Second, we focus on an SC-EDF whereas the work in [14] studied the SC service disciplines in a general fashion. We give a solid proof that the SC-EDF can make the SC service disciplines to achieve the network utilization achievable by the SC-EDF discipline. We also prove that the SC-EDF requires little overhead (\(O(1)\)) for deadline calculation.

The work in [18] studied a hierarchical link-sharing and priority service in the context of an SC service disciplines in a variable-sized packet environment. Although the focus of the work was different from ours, we base our work on it because of the common interest in the packet environment.

This paper is organized as follows. In Section II, we present the background needed to understand the SC-EDF discipline. In Section III, we focus on the SC-EDF and discuss network utilization and complexity for an deadline calculation. In Sec-
tion IV, we compare the SC service discipline using SC-EDF with the multi-rate service discipline. Lastly, Section V concludes this paper.

II. THE SERVICE CURVES

In Section II-A, we present our network modeling and the traffic specifications assumed in this paper. In Section II-B, we define SC’s and derive some analysis results applicable to SC’s in general. We use the same definition as in [18]. The presented analysis results are to some extent similar to those in [3]. However, they are different from those in [3] in that we handle variable-sized packet environment whereas the results in [3] are applicable only to fixed-sized environment. We review the SC service discipline in Section II-C. The SC-EDF, which is the focus of this paper, can be considered as a special case of the SC reviewed in the section.

A. Network modeling and traffic specifications

A network is modeled by a series of service disciplines. In the model, the transmission links are ignored for the convenience of discussion. We denote the input amount from session $i$ at a service discipline during the time interval $(s, t)$ by $R_{i}^{in}(s, t)$ and the output amount by $R_{i}^{out}(s, t)$. We define $R_{i}^{in}(s, t) = R_{i}^{out}(s, t) = 0$ if $s \geq t$. For notational convenience, we denote $R_{i}^{in}(0, t)$ and $R_{i}^{out}(0, t)$ by $r_{i}^{in}(t)$ and $r_{i}^{out}(t)$, respectively. When there are multiple service disciplines, we appropriately superscribe the notations to distinguish different service disciplines (e.g., $R_{i}^{out,m}(s, t)$, $r_{i}^{out,m}(t)$ for the output amounts from the $m$-th service discipline that session $i$ passes through). It is said that session $i$ is in a backlogged period in a service discipline if there exist any packets to serve from session $i$.

Each session must give a traffic specification to the network. A traffic specification is expressed by a traffic rate function $b_{i}(\cdot)$. Intuitively, $b_{i}(t)$ indicates the maximum amount of traffic that is allowed to be transmitted from session $i$ during a time interval of length $t$. Thus, $b_{i}(\cdot)$ must be a non-decreasing function and we say that $b_{i}(u) = 0$ for all $u \leq 0$. We say that session $i$ is $b_{i}$-smooth or traffic envelope function of session $i$ is $b_{i}(\cdot)$ if the incoming traffic amount from session $i$ to the network during the interval $(s, t)$ is not greater than $b_{i}(t-s)$. As a special case of $b_{i}$, we say that session $i$ is $(\sigma, \rho)$-smooth if $b_{i}(t) = \sigma + \rho t$. $\sigma$ represents the burstiness allowed to session $i$, $\rho$ is the average traffic rate.

We also say that session $i$ is $K(\sigma, \rho)$-smooth or traffic envelope function of session $i$ is $K(\sigma, \rho)$ if $b_{i}(t) = \min_{k=1,\ldots, K} \{\sigma^{k} + \rho^{k} t\}$. By a $K(\sigma, \rho)$ traffic envelope function, we can accurately specify the variability of a VBR video traffic amount according to intervals [13], [19]. See Figure 1. It illustrates a traffic specification when a session transmits an MPEG-coded video. An MPEG encoder produces three types of encoded frames: $I$ (Intracoded), $P$ (Predicted), and $B$ (Bidirectional). We can use the knowledge of the encoding pattern of frame sequences, $IBBPBB$ for example, and the largest frame sizes of $I$, $B$, and $P$ to specify the MPEG-coded video traffic. In the figure, the traffic envelope function is represented in two ways: $(\sigma, \rho)$-smooth and $3(\sigma, \rho)$-smooth. Note that when expressed as $3(\sigma, \rho)$-smooth, the specification can be much more accurate than when expressed as $(\sigma, \rho)$-smooth.

B. Some analysis results applicable to SC’s

A guaranteed SC for session $i$ can be derived when one or more service disciplines are given and we know the input and output traffic functions in advance. For the time being, we consider a service discipline in which the input and output functions are known. Suppose that there is a non-decreasing function $S_{i}(\cdot)$, $S_{i}(u) = 0$ for all $u \leq 0$ that satisfy the following: For any time $t$ at which a packet of a session $i$ departs the service discipline, there exists a time $s$, $s < t$, that is the beginning of one of session $i$’s backlogged periods such that:

$$r_{i}^{out}(t) \geq r_{i}^{in}(s) + S_{i}(t-s). \quad (1)$$

Then, we say that $S_{i}(\cdot)$ is guaranteed for session $i$ by the service discipline. Intuitively, $S_{i}(t)$ indicates the minimum service amount served for session $i$ by the service discipline during an interval $t$ starting from the beginning of a session $i$’s backlogged period. (1) can be equivalently expressed as follows: For any time $t$ at which a packet of a session $i$ departs the service discipline,

$$r_{i}^{out}(t) \geq \min_{s \in B_{i}(t)} \{r_{i}^{in}(s) + S_{i}(t-s)\} \quad (2)$$

where $B_{i}(t)$ is the set of the start time points of the session $i$’s backlogged periods that is not greater than the packet departure time $t$.

Now we consider multiple service disciplines which the session $i$ passes through. When we know the guaranteed SC for session $i$ in each of these service disciplines, we can derive a guaranteed SC for session $i$ by the multiple service disciplines. In other words, we can reduce the multiple service disciplines to one which guarantees the derived SC. Theorem I presents how to derive such an SC. (Theorems similar to the following three can be found in [3]. As mentioned earlier, they deal with different packet environments, namely, a fixed-sized vs. variable-sized packet environment.) In this paper, due to space constraints, we have omitted all the proofs which can be found in [12].

**Theorem 1:** Suppose that session $i$ passes through $M$ service disciplines in tandem and the $m$-th service discipline guarantees $S_{i}^{m}(\cdot)$ for session $i$. Then, the $M$ service disciplines
guarantee $S_i^{net}(t)$ for session $i$ where

$$S_i^{net}(t) = \min \{ |S_m^M|S_i^m(t_m) : t_m > 0 \text{ and } \sum_{m=1}^M t_m = t \}.$$  

(3)

When a service discipline guarantees an SC for session $i$, the delay and backlog bound to session $i$ can be derived.

**Theorem 2:** Let $b_i(\cdot)$ be the traffic envelope function of session $i$. When a service discipline guarantees $S_i(\cdot)$ for session $i$, the delay at the service discipline is less than

$$\max \{ S_i(k + \Delta) : \Delta > 0 \text{ and } b_i(k) < S_i(k + \Delta) \}.$$  

(4)

**Theorem 3:** Let $b_i(\cdot)$ be the traffic envelope function of session $i$. If a service discipline guarantees $S_i(\cdot)$ for session $i$, at any moment, the backlog is not greater than

$$\max \{ b_i(k) - S_i(k) \} + l^{max}.$$  

(5)

where $l^{max}$ is the maximum packet size.

Figure 2 graphically illustrates the delay and backlog bound for session $i$. In the figure, the maximum traffic amount from session $i$ and the minimum service amount to session $i$ during an interval $t$ are $b_i(t)$ and $S_i(t)$, respectively. It can be easily understood that the maximum horizontal distance from $b_i(\cdot)$ to $S_i(\cdot)$, denoted by $D(b_i||S_i)$, becomes the delay bound for session $i$ described in Theorem 2. Similarly, it can also be easily understood that the backlog bound described in Theorem 3 comes from the maximum vertical distance between $b_i(\cdot)$ and $S_i(\cdot)$. In the figure, at the interval $t_0$, the vertical distance becomes the maximum. In the theorem, the term $l^{max}$ is included in the backlog bound since each packet is cleared in the backlog after the last bit of the packet is served.

**C. The SC service discipline**

In Section II-B, we have presented that the delay and backlog bound at a service discipline can be derived if the service discipline guarantees an SC for a session. This section presents how to guarantee an SC for a session in a service discipline.

The SC service discipline allocates an SC to each session for guaranteeing an SC. Each incoming packet from a session is stamped a deadline from the allocated SC. The SC service discipline transmits a packet with the smallest deadline when the link is idle. Ties are broken arbitrarily. Transmissions can be either preemptive or non-preemptive. We consider the SC service discipline which transmits packets non-preemptively since a packet must be transmitted as a whole in a packet network environment.

Throughout this section, we denote the $k$-th packet from session $i$ by $p_i^k$. The arrival time, the departure time at the SC service discipline, and the length of the packet are denoted by $a_i^k$, $d_i^k$, and $l_i^k$ respectively. The start time of the $m$-th backlogged period of session $i$ is denoted by $b_i^m$.

Let’s see how to stamp deadlines in the SC service discipline. A deadline is assigned to each packet in such a way that if all the packets are transmitted before their deadlines, the allocated SC is guaranteed for each session. Suppose that a packet $p_i^k$ arrives during the $m$-th backlogged period of session $i$. Let $S_i(\cdot)$ be the allocated SC to the session and $D_i^m,k$ the deadline of the packet. The SC service discipline allocates the deadline $D_i^m,k$ as

$$D_i^m,k = \min \{ d : \min_{s \in B_i(b_i^m)} \{ r_i^m(s) + S_i(d - s) \} > \sum_{j=1}^k l_i^j \}.$$  

(6)

Note that $B_i(d_i^k) = B_i(b_i^m)$ since there exists no new backlogged period of session $i$ during the interval $[b_i^m, d_i^k]$. Thus, in (6), if $d_i^k < D_i^m,k$, then $r_i^m(s) + S_i(d_i^k - s) \leq \sum_{j=1}^k l_i^j = r_i^m$. Therefore, if each packet is transmitted before its deadline, $S_i(\cdot)$ is guaranteed for each session $i$.

For the computation of the deadlines in (6), the SC service discipline keeps a deadline curve $D_i(\cdot)$, which is defined as

$$D_i(t) = \min_{s \in B_i(t)} \{ r_i^m(s) + S_i(t - s) \}.$$  

(7)

$D_i(\cdot)$ is initialized to $S_i(\cdot)$ when session $i$ becomes backlogged for the first time. Subsequently, it is sufficient to update $D_i(\cdot)$ only when session $i$ becomes newly backlogged. It is updated as follows:

$$D_i(b_i^m,t) = \min \{ D_i(b_i^m - t, r_i^m(b_i^m) + S_i(t - b_i^m)) \}.$$  

(8)

Note that $D_i(b_i^m,t)$ in (8) is maintained only for $\{ t : S_i(t - b_i^m) > 0 \}$ as in [17] since this range is the only portion needed for subsequent deadline calculations. $(D_i(b_i^m,t))$ may be maintained for $t \geq b_i^m$ as in [18]. However, as discussed in Section III-B, the maintenance for this range can incur unnecessary complexity.) We define $D_i^{-1}(b_i^m,y)$ to be the smallest $x$ such that $D_i(b_i^m,x) = y$.

The deadline in (6) can be computed from $D_i^{-1}(\cdot)$. From (6),

$$D_i^m,k = \min \{ d : D_i(b_i^m,d) > \sum_{j=1}^k l_i^j \} = D_i^{-1}(b_i^m,\sum_{j=1}^k l_i^j) + l_i^k$$  

(9)

In (9), $\sum_{j=1}^k l_i^j$ refers to the next minimum unit that is greater than $\sum_{j=1}^k l_i^j$. Note that $\sum_{j=1}^k l_i^j > r_i^m$. This is the reason why we maintain $D_i(b_i^m,t)$ only for $\{ t : S_i(t - b_i^m) > 0 \}$ in (8).

Although we have mentioned that each packet needs to be transmitted before its deadline to guarantee an allocated SC, some may be transmitted after their deadlines since packets are transmitted non-preemptively. However, the departure time of each packet is still bounded. Theorem 4 tells that if the sum of the assigned SC’s doesn’t exceed the capacity, the delay of each packet is bounded. Note that Theorem 4 can also be used for admission control. (A similar theorem can be found in [18]. Theorem 4 differs in that the SC service discipline we consider
uses a different deadline curve. The deadline curve in [18] is more complex since it includes a hierarchical link-sharing.

Theorem 4: Consider the SC service discipline with a capacity \( r \) that serves \( N \) sessions. Let \( S_i(\cdot) \) be the assigned SC for each session \( i \). All the packets are served before their deadlines plus \( l^{\max}/r \) if

\[
\sum_{i=1}^{N} S_i(t) \leq rt
\]

where \( l^{\max} \) is the maximum packet size.

Now we derive a guaranteed SC when the SC service discipline assigns \( S_i(\cdot) \) to session \( i \). The SC service discipline guarantees \( S_i(t) \) such that

\[
\hat{S}_i(t) = \begin{cases} 
0, & t \leq l^{\max}/r \\
c_i(t - l^{\max}/r), & t > l^{\max}/r
\end{cases}
\]

for session \( i \) when the allocated SC is \( S_i(\cdot) \). Thus, whenever session \( i \) requires \( S_i(\cdot) \) as a guaranteed SC, the SC service discipline assigns \( S_i(\cdot) \) shifted left by \( l^{\max}/r \). We have omitted the detailed procedure deriving (11) due to space constraints. (See [12].)

Theorem 4 also applies to the preemptive packet environment. In this case, \( r^{\max}/r \) should be disregarded. (This can easily be understood by following the proof of the theorem.) That is, if packets are transmitted preemptively, they are all served before their deadlines. Thus, an assigned SC becomes a guaranteed SC in a preemptive packet environment.

III. SC-EDF

An SC-EDF is an SC with which an SC service discipline can achieve the same network utilization as the RC-EDF service disciplines. The SC-EDF for each session is constructed from an underlying RC-EDF service discipline. Suppose that, in the underlying RC-EDF service discipline, each session \( i \) has a regulator of the output function \( b_i(\cdot) \) and is associated a delay bound \( d_i \) at the EDF packet scheduler. In this case, the EDF packet scheduler guarantees the delay bound \( (d_i + l^{\max}/r) \) in non-preemptive packet environments where \( r \) is the link capacity. Let \( S_i(\cdot) \) be the SC-EDF for session \( i \). Then, \( S_i(\cdot) \) is constructed from \( b_i(\cdot) \) and \( d_i \) as following:

\[
S_i(t) = \begin{cases} 
0, & 0 \leq t \leq d_i \\
b_i(t - d_i), & t > d_i.
\end{cases}
\]

Typically \( b_i(\cdot) \) is specified by \( K(\sigma, \rho) \) functions, i.e., \( b_i(t) = \min_{k=1..K} \{ \sigma^k_i + \rho^k_i(t) \} \). We consider only such \( b_i(\cdot) \)'s for the remaining of the paper. In this case, \( S_i(\cdot) \) becomes the following:

\[
S_i(t) = \begin{cases} 
0, & 0 \leq t \leq d_i \\
\min_{k=1..K} \{ \sigma^k_i + \rho^k_i(t - d_i) \}, & t > d_i.
\end{cases}
\]

A. Conformance to RC-EDF service disciplines

This section presents how the SC service discipline with the SC-EDF’s can achieve the same network utilization as the RC-EDF service disciplines in two phases. First, we show that if a set of sessions is admitted by an underlying RC-EDF service discipline, the set is also admitted by the SC service discipline with the SC-EDF’s. Then, we show that the end-to-end delay bound to each session is the same when a session passes through the network composed by either of the two cases.

Let us see the first step in detail. If \( N \) sessions are admitted by an underlying RC-EDF service discipline, the following must be satisfied by admission control [5]:

\[
\sum_{i=1}^{N} b_i(t - d_i) + l^{\max} \leq rt \quad \text{for} \quad t \geq \min_{i=1..N} \{ d_i \}.
\]

We can reduce (14) as

\[
\sum_{i=1}^{N} b_i(t - d_i) \leq rt \quad \text{for} \quad t \geq \min_{i=1..N} \{ d_i \}.
\]

Let \( S_i(\cdot) \) be the SC-EDF for session \( i \). Then, from (15) and (12),

\[
\sum_{i=1}^{N} S_i(t) \leq rt \quad \text{for} \quad t \geq \min_{i=1..N} \{ d_i \}.
\]

Also, from (12),

\[
\sum_{i=1}^{N} S_i(t) \leq rt \quad \text{for} \quad t \leq \min_{i=1..N} \{ d_i \}.
\]

Thus, from (16) and (17), \( \sum_{i=1}^{N} S_i(t) \leq rt \) for any interval \( t \). Therefore, by Theorem 4, the \( N \) sessions are also admitted by the SC service discipline with the SC-EDF’s.

Now let us see the second phase. Consider the case that a session \( i \) passes through multiple RC-EDF service disciplines as shown in Figure 3. \( b_i(\cdot) \) is the output function of the regulator. Since there is no reason to use different output functions in the respective RC-EDF service disciplines [7], the same output function \( b_i(\cdot) \) is used at all the stages. The \( m \)-th EDF scheduler associates a delay bound \( d^m_i \) to the session. \( r^m \) is the \( m \)-th link capacity. Let \( d^\text{net,RC-EDF}_i \) be the end-to-end delay bound. Then, if the session is \( b_i^A \)-smooth, \( d^\text{net,RC-EDF}_i \) becomes the following [7]:

\[
d^\text{net,RC-EDF}_i = D(b^A || b_i) + \sum_{m=1}^{M} (d^m_i + \frac{l^{\max}}{r^{m}}).
\]

Figure 4 illustrates a sequence of SC service disciplines with the SC-EDF’s derived from the RC-EDF service disciplines in Figure 3. In the figure, \( S_i(\cdot) \) becomes the following:

\[
S^m_i(t) = \begin{cases} 
0, & 0 \leq t \leq d^m_i \\
b_i(t - d^m_i), & t > d^m_i.
\end{cases}
\]
Let \( \hat{S}_i^{\text{net}}(\cdot) \) be a guaranteed SC for the session at the \( m \)-th SC service discipline. By (11), \( \hat{S}_i^{\text{net}}(t) \) becomes the following:

\[
\hat{S}_i^{\text{net}}(t) = \begin{cases} 
0, & 0 \leq t \leq d_i^m + l^{\max}/r_m, \\
 b_i(t - d_i^m - l^{\max}/r_m), & t > d_i^m + l^{\max}/r_m.
\end{cases}
\] (20)

Then, by Theorem 1, the SC-EDF disciplines guarantee \( \hat{S}_i^{\text{net}}(\cdot) \) for the session where

\[
\hat{S}_i^{\text{net}}(t) = \begin{cases} 
0, & 0 \leq t \leq d_i^{\text{sum}} \\
 b_i(t - \sum_{m=1}^{M} (d_i^m + l^{\max}/r_m)), & t > d_i^{\text{sum}}.
\end{cases}
\] (21)

Let \( d_i^{\text{net,SC-EDF}} \) be the end-to-end delay bound. Then, if the session is \( b_i^A \)-smooth, by Theorem 2, \( d_i^{\text{net,SC-EDF}} \) becomes the following:

\[
d_i^{\text{net,SC-EDF}} = D(b_i^A||\hat{S}_i^{\text{net}}) = D(b_i^A||b_i) + \sum_{m=1}^{M} (d_i^m + l^{\max}/r_m).
\] (23)

From (18) and (23), the delay bounds to the session are identical.

### B. Deadline calculation

This section discusses the complexity of the deadline calculation when an SC-EDF is assigned to each session. The complexity mainly comes from updating the deadline curve.

We first try to give an intuition by a simple example in which SC-EDF \( S_i(\cdot) \) for session \( i \) is given as a simple linear curve as follows:

\[
S_i(t) = \begin{cases} 
0, & 0 \leq t \leq d_i \\
 \sigma + \rho(t - d_i), & t > d_i.
\end{cases}
\] (24)

The deadline curve \( D_i(\cdot) \) is initialized to \( S_i(\cdot) \) in \( O(1) \) complexity. Consider the case that \( D_i(\cdot) \) is secondly updated. \( D_i(b_i^m;\cdot) \) becomes the following:

\[
D_i(b_i^m; t) = \min\{S_i(t - b_i^m), S_i(t - b_i^m) + r_i^{\max}(b_i^m)\}.
\] (25)

In (25), we maintain \( D_i(b_i^m; t) \) for the time \( t \) such that \( t > b_i^m + d_i \). This is so since \( S_i(t - b_i^m) > 0 \) for this range. Figure 5 illustrates how to update \( D_i(b_i^m; t) \). \( D_i(b_i^m;\cdot) \) becomes linear after the time \( (b_i^m + d_i) \). Thus, \( D_i(b_i^m;\cdot) \) can be updated in \( O(1) \) complexity for this range. Inductively, it can be easily understood that \( D_i(b_i^m;\cdot) \) can also be updated in \( O(1) \).

(\text{Note that } D_i(b_i^m; t) \text{ is not linear for } t \geq b_i^m \text{. Thus, if we would maintain } D_i(b_i^m; t) \text{ for } t \geq b_i^m, \text{ updating } D_i(b_i^m;\cdot) \text{ could require extremely high complexity to remember the changing time points.})

The following theorem shows how to update the deadline curve \( D_i(\cdot) \) at each step. In the theorem, \( S_i(\cdot) \) is given as a K-piece concave linear curve after \( d_i \).

**Theorem 5**: Let the SC-EDF \( S_i(\cdot) \) for session \( i \) be

\[
S_i(t) = \begin{cases} 
0, & 0 \leq t \leq d_i \\
 \min_{k=1\cdots K} \{\sigma_i^k + \rho_i^k(t - d_i)\}, & t > d_i.
\end{cases}
\] (26)

Then, \( D_i(b_i^m;\cdot) \) becomes the following:

\[
D_i(b_i^m; t) = \min_{k=1\cdots K} \{C_i^{m,k} + \rho_i^k(t - d_i)\} \quad \text{for } t > b_i^m + d_i
\] (27)

where \( C_i^{k} = -\rho_i^k b_i^m \) and \( C_i^{m,k} = \min\{C_i^{m-1,k}, -\rho_i^k b_i^m + r_i^{\max}(b_i^m)\} \) for \( m \geq 2 \).

In Theorem 5, we maintain \( D_i(b_i^m; t) \) for \( t > b_i^m + d_i \) since \( S_i(t - b_i^m) > 0 \) for this range. \( D_i(b_i^m;\cdot) \) can be updated in \( O(K) \) since each \( C_i^{m,k} \) requires \( O(1) \) for \( k = 1 \cdots K \). Note \( D_i(b_i^m;\cdot) \) is a \( K \)-piece concave linear curve for this range. Thus, \( D_i^{-1}(b_i^m;\cdot) \) can be calculated in \( O(K) \). Since, in most cases, \( K \) is restricted to a small number, we can regard that the deadline calculation for an SC-EDF is done in a constant time.

### IV. COMPARISON OF THE SC-EDF WITH THE MULTI-RATE SERVICE DISCIPLINE

We compare the SC service discipline with the SC-EDF’s to the multi-rate service discipline in terms of network utilization and scalability.

As mentioned in Section I-A, the multi-rate service discipline is a GPS service discipline that guarantees a traffic specification characterized by multiple rates [13]. Each incoming packet of a session is given a time-stamp from the traffic rate. Packets are transmitted in an increasing order of the timestamps. Ties are broken arbitrarily. Originally, the multi-rate service discipline was proposed for ATM networks which belong to a fixed-sized packet environment. If the multi-rate service discipline is used in a variable-sized environment, transmissions can be either preemptive or non-preemptive. Only in this section, we assume that packets are transmitted preemptively to focus on the comparison.

To compare with the multi-rate service discipline, we set the SC-EDF \( S_i(\cdot) \) for session \( i \) as following:

\[
S_i(t) = \begin{cases} 
0, & 0 \leq t \leq d_i \\
 b_i(t - d_i), & t > d_i
\end{cases}
\] (28)

where \( b_i(\cdot) \) is the traffic rate function of the session in the multi-rate service discipline and \( d_i \) is the worst-case tolerable delay. Note that we can consider \( b_i(\cdot) \) as the output function of the regulator in the underlying RC-EDF service discipline.

As mentioned in Section II-A, \( b_i(\cdot) \) is a \( K(\sigma, \rho) \) function, i.e.,

\[
b_i(t) = \min_{k=1\cdots K} \{\sigma_i^k + \rho_i^k t\}.
\]
We compare network utilization first. We show, in two steps, that the SC service discipline with the SC-EDF’s can achieve strictly higher network utilization than the multi-rate service discipline. We first show that if a set of sessions is admitted by the multi-rate service discipline, the set is also admitted by the SC service discipline. Then, we show that there exists a set of sessions that are admitted by the SC service discipline, but cannot be admitted by the multi-rate service discipline.

Let us see the first step in detail. In the multi-rate service discipline, the end-to-end delay bound to a session is distributed to each service discipline that the session passes through as local delay bounds. Each session uses the traffic function and the distributed local delay bound at each service discipline. Thus, the admission of each session is localized to the respective service disciplines. Note that, in the case of a single service discipline, the SC service discipline with the SC-EDF’s has the highest network utilization since the RC-EDF service discipline is optimal in this case [6]. Therefore, we conclude that if a set of sessions is admitted by the network composed of the multi-rate service disciplines, the set is also admitted by the network with the SC service disciplines.

We now show the second step with an example. Consider two sessions, session 1 and session 2, that try to enter a service discipline. Both sessions have the same traffic rate, but different worst-case tolerable delays. Specifically, let \( b_i(t) = \min\{1 \text{ Mbt}, 30 \text{ Kb} + 400 \text{ Kbt}\} \) for \( i = 1, 2 \) and \( d_1 = 30 \text{ ms}, d_2 = 120 \text{ ms} \). We assume that the link capacity is 1 Mbt. Both sessions are admitted if the service discipline is the SC service discipline with the SC-EDF’s, since \( \sum_{i=1}^{2} b_i(t - d_i) \leq 1 \text{ Mbt} \). However, session 1 cannot be admitted by the multi-rate service discipline. The reason is as following. Since both sessions have the same traffic rate, the actual delay bounds to both sessions are identical. In this case, the bound is 55 ms if we follow the Lemma 3.3 presented in [13]. Therefore, the delay requirement to session 1 cannot be satisfied.

Now, let us turn to the comparison of scalability. The multi-rate service discipline requires \( O(K) \) complexity to compute the time-stamps of each packet. In addition, \( O(\log N) \) complexity is required for maintaining a priority queue where \( N \) is the number of packets in the queue. As mentioned in Section III-B, the SC service discipline requires \( O(K) \) complexity to compute the deadline of each packet. The SC service discipline can maintain a priority queue either for the head packets from each session or for all the packets from all sessions. We consider the case that the SC service discipline maintains one queue for all the packets to compare with the multi-rate service discipline. In this case, the SC service discipline requires \( O(\log N) \) complexity for the priority queue where \( N \) is the number of packets in the queue. Therefore, both have the same scalability.

V. CONCLUSION

In this paper, we propose the SC service discipline with SC-EDF’s for packet networks. It achieves very high network utilization. (It can achieve the same utilization as the RC-EDF service discipline.) It has \( O(K) \) deadline calculation complexity if the traffic rate function of each session is a \( K(\sigma, \rho) \) function. Different from the RC-EDF service disciplines, it doesn’t need regulators at all. Thus, it has better scalability than the RC service disciplines and is work-conserving. Compared to the GPS service disciplines including the multi-rate service discipline, it can achieve strictly higher network utilization.

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